

## Stoner Model

### 1. Stoner Criterion Derivation

Derive the Stoner criterion for the onset of ferromagnetism. Begin by considering the expression for the total energy of an electron gas and introduce the exchange interaction.

*Hint:* Use the expression for the density of states  $D(E)$  and the exchange interaction parameter  $U$ . Show that ferromagnetism occurs when:

$$\frac{1}{2}UD(E_F) > 1$$

### 2. Density of States Calculation

For a free electron gas in three dimensions, the density of states at the Fermi energy  $E_F$  is given by:

$$D(E_F) = \frac{3n}{2E_F}$$

where  $n$  is the electron density. Using this, express the Stoner criterion in terms of  $n$ ,  $E_F$ , and  $U$ .

## Exercise 2: Exchange Interaction and external Field

How does a metal with ferromagnetic exchange interaction react to the presence of an external field  $B_{ext}$ . Derive the magnetization and susceptibility.

Consider a system that does not satisfies the Stoner criterion. How does the exchange interaction alter the paramagnetic behavior.

$$\chi_{Pauli} = \mu_B^2 D(E_F)$$

## Exercise 3: Band Structure Effects

### 1. Band Structure and Stoner Criterion

Explain how the shape of the density of states  $D(E)$  near the Fermi level influences the likelihood of meeting the Stoner criterion in a material with a non-free electron band structure.

### 2. Stoner Model and Transition Metals

Transition metals often exhibit ferromagnetism. Choose one transition metal (e.g., iron, cobalt, or nickel) and discuss how its band structure contributes to its ferromagnetic properties in the context of the Stoner model.

## Exercise 4: Numerical Problems

### 1. Numerical Calculation

Given the following parameters for Fe and Pd:

- Density of States  $D(E_F) = 3.1 \text{ 1/eV}$  and  $2.2 \text{ 1/eV}$
- Exchange interaction parameter  $I = 0.84 \text{ eV}$  and  $0.76 \text{ eV}$

Calculate whether this metal will exhibit ferromagnetism according to the Stoner criterion.

### 2. Critical Exchange Interaction

For Pd in the previous problem, determine the critical value of the exchange interaction parameter  $U_c$  necessary for the onset of ferromagnetism.

## Exercise 5: Advanced Concepts

Discuss some limitations of the Stoner model. What are some of the physical effects or interactions that are not accounted for in this model that could affect the magnetic properties of a material?